Technical Note on Design of Suspension Parameters for FSAE Vehicle

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ABSTRACT: Correct suspension parameters determination is one of the most important design issues in the development of each type of car. The aim of the suspension design in the field of race cars is to provide ideal operating conditions for the tire and to allow it to generate the maximum amount of traction, braking and lateral forces which determine a vehicle's acceleration capabilities. This article describes the determination of the Formula Student/SAE car suspension parameters related to the vertical dynamics of the car as a basic point in tuning up the suspension on the car itself in real operating conditions.

KEYWORDS: Suspension parameters, spring rate, damping rate, Formula Student/SAE.

1 INTRODUCTION

Suspension is one of the most important pieces of equipment on each car. It has different functions: it carries all the vehicle's loads; maintains the correct wheel alignment to the ground; reduces the effect of shock forces when passing ground disturbances; controls the vehicle's longitudinal and lateral speed, and maintains the tire contact patch in contact with the ground for the maximum time possible. The mentioned requirements are provided by different suspensions parts divided into the guiding elements and force generating elements. In the following articles, the procedure for the determination of the spring rate and damping rate is presented. The numerical values of the mentioned constants are computed for a Formula Student/SAE car and later used when building the real Formula Student/SAE car at CULS Prague.

2 DETERMINATION OF SPRING RATES

The described approach for the determination of all the necessary suspension parameters related to the vertical dynamics is based on a quarter-sized car model. The basic points for the suspension design parameters are the mass properties of the vehicle (see tab.1).

Table 1: Vehicle mass properties.

Constant	Value	Unit	Signification
m	300	[kg]	overall vehicle mass
w_F / w_R	45 / 55	[% / %]	mass distribution related to front / rear axis
$m_F = w_F m$	135	[kg]	overall mass on front axis
$m_R = w_R m$	165	[kg]	overall mass on rear axis
m_{uF}	27.78	[kg]	overall unsprung mass on front axis
m_{uR}	29.11	[kg]	overall unsprung mass on rear axis
$m_{sF} = m_F - m_{UF}$	107.22	[kg]	overall sprung mass on front axis
$m_{sR} = m_R - m_{UR}$	135.89	[kg]	overall sprung mass on rear

According the suggestions from literature (Milliken & Milliken 1995) for low-downforced racing cars, the initial choice of ride frequencies is as follow: front ride frequency f_{nF} = 2.1 Hz, rear ride frequency, f_{nR} = 1.9 Hz Then ride rates for front K_{rF} and rear K_{rF} end of the vehicle, with respect to corner (either left or right which equals).

$$f_{nF,R} = \frac{1}{2\pi}\sqrt{\frac{K_{rF,R}}{\frac{m_{sF,R}}{2}}}, \qquad K_{rF,R} = (2\pi f_{nF,R})^2 \frac{m_{sF,R}}{2}$$

$$K_{rF} = (2\pi.2,1)^2 \frac{107,22}{2} = 9333,67 Nm^{-1} \qquad K_{rR} = (2\pi.1,9)^2 \frac{135,89}{2} = 9683,61 Nm^{-1}$$

With spring rate $K_t = 125000 Nm^{-1}$ of chosen tire Hoosier 20x7.5x13 - pressure 14 PSI (Honzík , 2008)

$$K_{wF,R} = \frac{K_{rF,R} K_t}{K_t - K_{rF,R}}$$

$$K_{wF} = \frac{9333,67.125000}{125000 - 9333,67} = 10086,84 Nm^{-1} \qquad K_{wR} = \frac{9683,61.125000}{125000 - 9683,61} = 10496,78 Nm^{-1}$$

Final real spring rates $K_{sF,R}$ must be recalculated using the so-called "installation ratio" IR (Milliken & Milliken, 1995) defined as rate of change of spring compression with wheel movement. To slightly simplify the non-linear function for pull-rod type suspension, installation ratios have to be dealt with as a constant $IR_F = IR_F(0) = 1{,}5$ and $IR_R = IR_R(0) = 1{,}4$. Then

$$K_{sF} = \frac{K_{wF}}{IR_F^2} = \frac{10086{,}84}{1{,}5^2} = 4483{,}04 Nm^{-1} \qquad K_{sR} = \frac{K_{wR}}{IR_R^2} = \frac{10496{,}78}{1{,}4^2} = 5355{,}5 Nm^{-1}$$

3 CALCULATION OF ANTI-ROLL BAR PARAMETERS FOR DESIRED ROLL GRADIENTS

Roll gradient $RG[deg/g]$ gives information on how much the body rolls due to the lateral acceleration of the whole car. The desired set up is up to 1.5° / 1g, referred to by suggestions given in (Milliken & Milliken, 1995) as the Formula Student/SAE car achieved a max. lateral acceleration of about 1.5g. At first, roll stiffness is computed using front and rear track ($t_F = 1{,}230m$, $t_R = 1{,}205m$), spring rates $K_{wF,R}$.

$$K_{\varphi F} = \frac{1}{2} K_{wF} t_F{}^2 = 0{,}5.10086{,}84.1{,}230^2 = 7630{,}19 \frac{Nm}{rad} = 133{,}17 \frac{Nm}{deg}$$

$$K_{\varphi R} = \frac{1}{2} K_{wR} t_R{}^2 = 0{,}5.10496{,}78.1{,}205^2 = 7620{,}8 \frac{Nm}{rad} = 133{,}01 \frac{Nm}{deg}$$

The next step in the determination of anti-roll bars is the computation (Milliken & Milliken, 1995) of the height of the center of gravity of the sprung mass h_S , sprung mass distribution a_S and rolling moment lever arm h_{RM} (with the help of used variables: height of the center of gravity of the whole car $h = 0{,}38m$, wheel radius $r_F = 0{,}26m$, $r_R = 0{,}26m$, and front / rear roll center heights $z_F = 0{,}04m$, $z_R = 0{,}06m$).

$$h_s = \frac{mh - m_{uF} r_F - m_{uR} r_R}{(m_{sF} + m_{sR})} = \frac{300.0{,}38 - 27{,}78.0{,}26 - 29{,}11.0{,}26}{107{,}22 + 135{,}89} = 0{,}371m$$

$$a_S = \frac{m_{sF}}{m_{sF} + m_{sR}} = \frac{107{,}22}{107{,}22 + 135{,}89} = 0{,}44$$

$$h_{RM} = h_S - [z_F - (z_R - z_F)(1 - a_S)] = 0{,}347 - [0{,}04 - (0{,}06 - 0{,}04) * (1 - 0{,}44)] = 0{,}319m$$

For anti-roll bars stiffness $K_{\varphi B}$, the calculation of the rolling moment per 1g of lateral acceleration, M_{φ} / A_{y} and the computation of the overall desired roll stiffness K_{φ} is required.

$$\frac{M_{\varphi}}{A_{y}} = h_{RM}\left(m_{sF} + msR\right)g = 0{,}319.\left(107{,}22 + 135{,}89\right).9{,}81 = 762{,}9Nm$$

$$K_{\varphi} = \frac{M_{\varphi} / A_{y}}{RG} = \frac{762{,}9}{1.5} = 508{,}6N.m / deg$$

$$K_{\varphi B} = K_{\varphi} - K_{\varphi F} - K_{\varphi R} = 508{,}6 - 133{,}17 - 133{,}01 = 242{,}42Nm / deg$$

The recommendation (Milliken & Milliken, 1995) is to start with a total lateral load distribution to be 5% more than the weight distribution wF at the front axle. Based on this fact, the required anti-roll bar stiffness for the front and rear axle $K_{\varphi B\,F,R}$ is determined from the overall desired roll stiffness K_{φ} as follows

$$K_{\varphi B\,F} = K_{\varphi}\ .\ (\frac{w_F + 5}{100})\ -\ K_{\varphi F} = 508.6\ .\ (\frac{45 + 5}{100}) - 133.17 = 121.13 \quad Nm / deg$$

$$K_{\varphi B\,R} = K_{\varphi B}\ -\ K_{\varphi B\,F} = 242.42 - 121.13 = 121.29 \quad Nm / deg$$

Because the anti-roll bar installation ratio $IR_{AB\,F,R}$ (the rate of anti-roll bar displacement / roll with body roll) is expected to be the same as the ratio for the springs $IR_{AB\,F} = IR_{AB\,F}(0) = 1{,}5$ and $IR_{AB\,R} = IR_{AB\,R}(0) = 1{,}4$, then the final front and rear anti-roll bar stiffness $K_{\varphi AB\,F,R}$ is :

$$K_{\varphi AB\,F} = \frac{K_{\varphi B\,F}}{IR^2_{AB\,F}} = \frac{121{,}13}{1{,}5^2} = 53{,}83 \ Nm / deg$$

$$K_{\varphi AB\,F} = \frac{K_{\varphi B\,R}}{IR^2_{AB\,R}} = \frac{121{,}29}{1{,}4^2} = 61{,}88 \ Nm / deg$$

4 DETERMINATION OF DAMPING COEFFICIENTS

The baseline mean ride damping coefficients for each wheel of the front C_{brF} and rear axle C_{brR} result from the critical damping values $C_{brFcrit}$, $C_{brRcrit}$ (the critical damping coefficients of the sprung mass) multiplied by the recommended (Milliken & Milliken, 1995) damping ratios for the front and rear axle $\zeta_F = 0{,}4$, resp. $\zeta_R = 0{,}45$.

$$C_{brFcrit} = 2\sqrt{\frac{m_{sF}}{2}K_{wF}} = 2\sqrt{\left(\frac{107{,}22}{2}10008{,}84\right)} = 1470{,}73\frac{Ns}{m}$$

$$C_{brRcrit} = 2\sqrt{\frac{m_{sR}}{2}K_{wR}} = 2\sqrt{\left(\frac{135{,}89}{2}10496{,}78\right)} = 1689{,}05\frac{Ns}{m}$$

$$C_{brF} = \zeta_F C_{brFcrit} = 0{,}4.1470{,}73 = 588{,}29\frac{Ns}{m}$$

$$C_{brR} = \zeta_R C_{brRcrit} = 0{,}45.1689{,}05 = 760{,}07\frac{Ns}{m}$$

To obtain the final values of the mean damping coefficients set-up on the dampers, $C_{F,R}$, these must be corrected by the corresponding installation ratio again

$$C_F = \frac{C_{brF}}{IR^2} = \frac{588{,}29}{1{,}5^2} = 261{,}46\frac{Ns}{m}\ , \qquad C_R = \frac{C_{brR}}{IR^2} = \frac{760{,}07}{1{,}4^2} = 387{,}79\frac{Ns}{m}$$

For better control of resonance and the energy released by the spring, more damping force is required by the damper during the rebound (bilinear model). This asymmetry for compression C_C and extension C_E damping is expressed by the compression/extension ratio $R_{CE} = \frac{C_C}{C_E}$ with a typically value 0.4 as recommended from (Dixon, 1999).

Then, the modified damping coefficients, as a starting point for the next suspension tuning for the linear progressivity of compression and extension, are calculated for both axles as follows

$$C_{EF} = \frac{2\,C_F}{1+R_{CE}} = \frac{2\,.261{,}46}{1+0{,}4} = 373.51\ \frac{Ns}{m}, \quad C_{CF} = R_{CE}\,C_{EF} = 149.41\ \frac{Ns}{m}$$

$$C_{ER} = \frac{2\,C_R}{1+R_{CE}} = \frac{2\,.387{,}79}{1+0{,}4} = 553.99\ \frac{Ns}{m}, \quad C_{CR} = R_{CE}\,C_{ER} = 221.59\ \frac{Ns}{m}$$

5 CONCLUSIONS

This paper presents an approach for the determination of basic suspension parameters - spring stiffness, anti-roll bar stiffness and damping coefficients. The approach is based on linear vibrations dynamics and semi-experimental recommendations for the choice of basic constants. The presented approach can be applied for any road racing vehicle.

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REFERENCES

Milliken, W., Milliken D. , 1995. *Race Car Vehicle Dynamics.* SAE, 1995.

Honzík T., 2008. *Front Axle Design for Formula SAE*. VUT Brno

Rill, G. : *Vehicle dynamics. Lecture notes, October 2007*. Fachhochshule Regensburg.

Available on http://homepages.fhregensburg.de/~rig39165/skripte/Vehicle_Dynamics.pdf

Dixon, J. : 1999 , *The Shock Absorbers Handbook* , SAE 1999

Index of Titles

Volume 3/ 2010